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Particle models from orientifolds at Gepner-orbifold points

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Abstract

We consider configurations of stacks of orientifold planes and D-branes wrapped on a non trivial internal space of the structure $(Gepner\ model)^{c=3n} \times \mathbf{T^{2(3-n)}}/\mathbf{Z_N}$, for $n = 1, 2, 3$. By performing simple moddings by discrete symmetries of Gepner models at orientifold points, consistent with a $\mathbf{Z_N}$ orbifold action, we show that projection on D-brane configurations can be achieved, generically leading to chiral gauge theories. Either supersymmetric or non-supersymmetric (tachyon free) models can be obtained. We illustrate the procedure through some explicit examples.

1 Introduction

The identification of D-branes [1], as an essential ingredient of the string theory structure, lies at the centre of the radical change that lead to the present conception of string theory. From a phenomenological approach, the fact that gauge interactions are confined on brane world volumes, has opened new perspectives for understanding the ways in which the Standard Model of fundamental interactions can be embedded in string theory. A key issue in this context is the origin of chirality. It is known that either branes intersecting at angles [2–12] (or equivalently in the presence of turned on fluxes) or branes stuck at singularities [13–18] (or a combination of both [6]) can lead to chiral fermions[1]. Concrete constructions of particle models on D-brane worlds have been performed, mainly, in the presence of toroidal like background compactifications of Type II A/B string theory. In particular, in the context of Type II theory orientifolds D-branes are required to provide the RR charges to cancel orientifold plane ones. Toroidal like compactifications are, generically, easy to handle and allow for a simple description in geometrical terms. Nevertheless, the possible space of string backgrounds is much more richer and deserves further investigation. Relevant steps towards the understanding of the algebraic and geometrical interpretation of D-branes on Calabi-Yau manifolds have been performed (see, for instance [20–25]). Particularly appealing, in a first step approach towards generic non flat background descriptions, are the, so called, $N = 2$ coset theories, where the internal space can be described in terms of solvable $N = 2$ conformal theories [26, 27]. In Gepner models, on which we concentrate in this article, such internal sector is spanned by a tensor product of $N = 2$ minimal conformal theories. Each minimal model is characterized by an affine level k implying a central charge $c = \frac{3k}{2+k}$. Blocks central charges must add up to total internal charge $c_{int} = 9$ in order to ensure conformal anomaly cancellation.

Extensive studies on such backgrounds have been performed in perturbative heterotic string and a growing understanding of type II orientifold theories on Gepner points has been acquired more recently [28–34].

From a phenomenological perspective two *minimum* non trivial requirements for a given string compactification to be acceptable are:

i.The gauge group must be big enough, in order to fit the standard model.

ii.Chiral fermions should be present.

Gepner orientifold constructions tend to lead to rank reduction, due to (see [35]

[1]An alternative proposal was made in [19] where turned on RR and NS fluxes could require chiral fermions

and references therein) discrete NS B field turned on (also understandable from the presence of different kinds of orientifold planes [36]). For instance, the so called ($D = 4$) $(k = 1)^9$ model, a parent Z_3 orbifold, possesses rank 2 gauge groups. Presence of higher rank, orthogonal groups, was first advanced in [30], where examples were given, and further analysed in [29](see also [30–33]). Actually, high values of k do generically lead to several tadpole equations which allow for higher ranks.

The only groups present in orientifold models, with complex representations (required for chirality), are unitary groups. However, in Gepner orientifolds, unitary groups do not always appear. For instance, for odd k level and diagonal invariant couplings (i.e. models with B type branes), in arbitrary dimensions, it is known (see i.e. [31] and index formula below) that, only orthogonal or symplectic groups can appear. On the other hand, in explicit simple model examples, where some minimal blocks are coupled through charge conjugate invariant, unitary groups have been found [29]. However, going beyond diagonal invariants, for instance by using the charge conjugate invariant, generically leads to a high number of tadpole equations which can be very difficult to tackle in concrete computations. Promising models have been constructed in [31] by projecting by simple current symmetries leading to a reduction in the number of tadpole equations. For even values of the level k unitary groups are known to appear (see [15, 29, 32, 33]. Such cases have some extra complications due to the presence of long and short orbits.

In [29] a general description of modding by minimal model phase symmetries in Gepner orientifold was provided.

In the present article we show that, by embedding these phase symmetry transformations of internal blocks into a Z_N action on Chan Paton factors, in a very similar way like an orbifold action on internal coordinates is embedded in gauge degrees of freedom, unitary groups with chiral matter can be easily obtained. In practice, it proves useful to start with diagonal invariant couplings which, as mentioned, are much easier to handle since they lead to a minimum number of tadpole cancellation equations. Thus, once solutions for such models have been found, leading to orthogonal or symplectic gauge theories for k odd, chiral theories are obtained by phase projecting. From the point of view of tadpole cancellation, modding leads to an increase in the number of equations to be solved due to the appearance of massless twisted RR states in the transverse channel. Interestingly enough, by knowing the form of a definite modding, a certain control on such number can be maintained [2]. Moreover, tadpole equations for modded

[2] in a somewhat opposite way to [31] where projections are used to reduce the number of equations

theory can be rather easily obtained from non-modded one.

It appears natural to think in terms of internal spaces where some complex planes are fulfilled by a Gepner model while the others are compactified on a torus. In such a scheme, stacks of orientifold planes and D-branes will wrap on non trivial cycles on the Gepner planes whereas parallel branes are expected to appear on the $C^{(3-n)}$ planes. We consider such situations in this paper. In particular, we show that, phase modding on Gepner sector can be accordingly accompanied by an orbifold projection on the torus leading again to chiral theories. Schematically the internal sector would be described by $(Gepner\ model)^{c=3n} \times \mathbf{T^{2(3-n)}}/\mathbf{Z_N}$, for $n = 1, 2, 3$. A rather similar idea was presented in [6] with D-branes intersecting at angles in the C^n plane but parallel on the remaining internal directions. An advantage of this approach is that, gauge group rank can be reduced, due to presence of Gepner term, but remain big enough in order to lead to phenomenologically interesting constructions.

The paper is organized as follows. Section 2 contains a generic introduction to Type IIB orientifold ideas. In Section 3 orbifold and Gepner like internal spaces, as well as hybrid orbifold-Gepner cases, are briefly discussed. Modding by phase symmetries is introduced and the construction of the corresponding supersymmetric projected characters is shown. Notation and general results closely follow those of reference [29]. The open string sector is discussed in section 4. Special emphasis is given to the way in which phase moddings are embedded as Chan-Paton factor twists. An index formula is presented. Tadpole cancellation is addressed in section 5. In section 6 we show, through some simple examples, how chiral models can be easily obtained in both supersymmetric and non supersymmetric models. Details are left to the Appendix.

Note: While this work was under completion we become aware of the nice results of Ref. [33]. Even if the two approaches are different there is still some overlap.

2 Type II orientifolds

In this section we briefly review the basic steps implied in the construction of orientifold models. Essentially an orientifold model is obtained by dividing out the orientation reversal symmetry of Type II string theory. Schematically, Type IIB torus partition function is defined as

$$\mathcal{Z}_T(\tau,\bar\tau) = \sum_{a,b} \chi_a(\tau)\mathcal{N}^{ab}\bar\chi_b(\bar\tau) \tag{2.1}$$

where the characters $\chi_a(\tau) = \mathrm{Tr}_{\mathcal{H}a} q^{L_0-\frac{c}{24}}$, with $q = e^{2i\pi\tau}$, span a representation of the modular group of the torus generated by S: $\tau \to -\frac{1}{\tau}$ and T: $\tau \to \tau + 1$ transforma-

tions. $\mathcal{H}_a$ is the Hilbert space of a conformal field theory with central charge $c = 15$ generated from a conformal primary state ϕ_a (similarly for the right moving algebra). In particular $\chi_a(-\frac{1}{\tau}) = S_{aa'}\chi_{a'}(\tau)$ and modular invariance requires $S\mathcal{N}S^{-1} = \mathcal{N}$(for left -right symmetric theories $\mathcal{N}^{ab} = \mathcal{N}^{ba}$). Generically, the characters can be split into a spacetime piece, contributing with $c_{st} = \bar{c}_{st} = \frac{3}{2}D$ and an internal sector with $c_{int} = \bar{c}_{int} = \frac{3}{2}(10 - D)$.

Let Ω be the reversing order (orientifolding) operator permuting right and left movers. Modding by order reversal symmetry is then implemented by introducing the projection operator $\frac{1}{2}(1+\Omega)$ into the torus partition function. The resulting vacuum amplitude reads

$$\mathcal{Z}_\Omega(\tau,\bar{\tau}) = \mathcal{Z}_T(\tau,\bar{\tau}) + \mathcal{Z}_K(\tau-\bar{\tau}). \tag{2.2}$$

The first contribution is just the symmetrization (or anti-symmetrization in case states anticommute) of left and right sector contributions indicating that two states differing in a left-right ordering must be counted once. The second term is the Klein bottle contribution and takes into account states that are exactly the same in both sectors. In such case, the operator $e^{2i\pi\tau L_0}e^{-2i\pi\bar{\tau}\bar{L}_0}$, when acting on the same states, becomes $e^{2i\pi 2it_K L_0}$ with $\tau - \bar{\tau} = 2it_K$ and thus

$$\mathcal{Z}_K(2it_K) = \frac{1}{2}\sum_a \mathcal{K}^a\chi_a(2it_K), \tag{2.3}$$

where $|\mathcal{K}^a| = \mathcal{N}^{aa}$. The Klein bottle amplitude in the *transverse channel* is obtained by performing an S modular transformation such that

$$\tilde{\mathcal{Z}}_K(il) = \frac{1}{2}\sum_a O_a^2\chi_a(il) \tag{2.4}$$

with $l = \frac{1}{2t_K}$ and

$$O_a^2 = 2^D\mathcal{K}^b S_{ba} \tag{2.5}$$

This notation for the closed channel coefficients highlights the fact that the Klein bottle transverse channel represents a closed string propagating between two crosscaps (orientifold planes) which act like boundaries. When integrated over the tube length, such amplitude leads, for massless states, to tadpole like divergences. In particular, for RR massless states such tadpoles must be cancelled for the theory to be consistent. Notice that, for such fields, O_a represents the charge of the orientifold plane (crosscap) under them and, therefore, inclusion of an open string sector with D-branes carrying $-O_a$ RR charge provides a way for having a consistent theory [1, 37, 38] with net vanishing charge. An open string cylinder amplitude, representing strings propagating

between two D-branes, and a Möbius strip amplitude with strings propagating between orientifold planes and D-branes must be included. In the long tube limit the sum of the contributions from the Klein bottle, cylinder and Möbius strip in the transverse channel must then factorize as

$$\tilde{\mathcal{Z}}_K(il) + \tilde{\mathcal{Z}}_M(il) + \tilde{\mathcal{Z}}_C(il) \to \sum_a (O_a + D_a)^2 \frac{1}{{m_a}^2} = \sum_a (O_a^2 + 2O_a D_a + D_a^2) \frac{1}{{m_a}^2} \quad (2.6)$$

where m_a is the mass of the state in χ_a. For massless RR fields D_a is the D-brane RR charge and absence of divergences requires

$$O_a + D_a = 0. \quad (2.7)$$

Cylinder amplitude in the direct channel should read

$$\mathcal{Z}_C(it_C) = \frac{1}{2} \sum_a \mathcal{C}_a \chi_a(it_C), \quad (2.8)$$

where

$$\mathcal{C}_a = C_{jka} n_j n_k \quad (2.9)$$

represents the multiplicity of states contained in $\chi_a(it)$ and n_j, n_k are Chan-Paton multiplicities. Namely, open sector states are of the form

$$|\Phi_k; i, j\rangle \lambda^k_{ji} \quad (2.10)$$

where Φ_k is a world sheet conformal field and j (i) label the type of branes where the string endpoints must be attached. n_j is the number of branes on each stack while λ^k_{ji} represent the corresponding wave functions in this brane basis expansion. C_{ija} are positive integers (actually $C_{ija} = 0, 1, 2$) generated when trace over open states $|\Phi_k; i, j\rangle$ is computed. When rewriting 2.8 in the transverse channel we recover $\tilde{\mathcal{Z}}_C(il)$ in 2.6 where with $D_a = D_{ja} n_j$ and

$$(D_{ja} n_j)^2 = \mathcal{C}_b S_{ba} = C_{jkb} n_j n_k S_{ba} \quad (2.11)$$

The Möbius strip amplitude is constructed in a similar way. However, since characters $\mathrm{Tr}_{\mathcal{H}_a}(e^{\pi i t(L_0 - \frac{c}{24})}\Omega) = \chi_a(it_M + \frac{1}{2})$ are non-real, it proves convenient to work in terms of the real "hatted" $\hat{\chi}_a(il + \frac{1}{2}) = e^{i\pi(h_a - c/24)} \chi_a(it_M + \frac{1}{2})$ characters. Thus, MS amplitude in the direct channel takes the form

$$\mathcal{Z}_M(it_M) = \frac{1}{2} \sum_a \mathcal{M}_a \hat{\chi}_a(it_M + \frac{1}{2}) \quad (2.12)$$

where now

$$\mathcal{M}_a = M_{ja} n_j \tag{2.13}$$

are integer numbers. The characters in the direct and transverse channels of the Möbius strip are related by the transformation [45] P: $it_M + \frac{1}{2} \to \frac{i}{4t_M} + \frac{1}{2}$ generated from the modular transformations S and T as $\mathsf{P} = \mathsf{T}\mathsf{S}\mathsf{T}^2\mathsf{S}$. Thus, we can rewrite the transverse channel in 2.6 representing a closed string propagating between a D-brane and an orientifold plane from where we read

$$O_a(D_{ja} n_j) = 2^{\frac{D}{2}} \mathcal{M}_b P_{ba} = 2^{\frac{D}{2}} M_{jb} n_j P_{ba} \tag{2.14}$$

In principle, once the Klein bottle partition function is obtained from the left-right symmetric type IIB torus partition function, a full consistent open string theory can be constructed by ensuring factorization, massless RR tadpole cancellation, and consistency restrictions on the integer coefficients C_{jia} and M_{ja}. Certainly, such steps can be more or less cumbersome depending on the type of models considered.

3 $D = 4$ Type IIB orientifolds on Gepner models and orbifolds

In $D = 4$ dimensions each moving sector of Type IIB theory is described by a conformal theory of total central charge $c_{tot} = c_{st} + c_{int} = 12$, where $c_{st} = 3$ and $c_{int} = 9$ are the central charges corresponding to space-time and internal (six dimensional) sectors, respectively. In a toroidal like compactification six free bosonic and fermionic fields, each one contributing with 1 and 1/2 units to central charge respectively, curl up the extra six dimensions in order to provide a consistent theory. Such compactifications are generically non chiral, since too many supersymmetries are preserved. If orbifold like singularities are present some or all of supersymmetry generators are projected out. For instance, consider $\mathbf{Z_N}$ orbifold action performed by the generator θ such that

$$\theta^x Y_i = \mathrm{e}^{2i\pi x v_i} Y_i \tag{3.1}$$

with x an integer number and where Y_I, $I = 1, 2, 3$ are complex bosonic coordinates parameterizing the $\mathbf{T^6}$ internal torus. The twist vector $v = (v_1, v_2, v_3)$ encodes the orbifold action on each complex plane. Thus, for instance, for untwisted massless Left (or Right) Ramond states of the form $|\sigma_0\sigma_1\sigma_2\sigma_3\rangle$ with $\sigma_0, \sigma_i = \pm\frac{1}{2}$, we have

$$\theta^x |\sigma_0\sigma_1\sigma_2\sigma_3\rangle = \mathrm{e}^{2i\pi x v\cdot\sigma} |\sigma_0\sigma_1\sigma_2\sigma_3\rangle \tag{3.2}$$

The invariance condition

$$\sigma_I v_I \in \mathbb{Z} \tag{3.3}$$

projects some of the fermionic states out and therefore reduces the number of supersymmetries.

In particular, the condition $\pm v_1 \pm v_2 \pm v_3 = 0$ ensures that there is a gravitino in both the NS-R and R-NS type IIB untwisted sectors. Projection under Ω, produces the closed sector of the orientifold and then leads to N=1, D=4 supersymmetry. Partition function can be found, for instance, in Ref. [16].

Gepner models [26] offer an alternative in which supersymmetric string vacua are provided in terms of an explicit algebraic construction. The internal sector is given by a tensor product of r copies of N=2 superconformal minimal models with levels k_j, $j = 1, ..., r$ and central charge

$$c = \frac{3k}{k+2} \quad , \quad k = 1, 2, ... \tag{3.4}$$

with total central charge $\sum_{j=1}^{r} c_{k_j}^{int} = 9$. Spacetime supersymmetry and modular invariance are implemented by keeping in the spectrum only states for which the total $U(1)$ charge is an odd integer. More explicitely, $N = 2$ minimal models unitary representations, are encoded in primary fields labelled by three integers (l, q, s) such that $l = 0, 1, ..., k$; $l + q + s = 0$ mod 2. They belong to the NS or R sector when $l + q$ is even or odd respectively. The conformal dimensions and charges of the highest weight states are given by

$$\Delta_{l,q,s} = \frac{l(l+2) - q^2}{4(k+2)} + \frac{s^2}{8} \mod 1 \tag{3.5}$$

$$Q_{l,q,s} = -\frac{q}{k+2} + \frac{s}{2} \mod 2. \tag{3.6}$$

Two representations labelled by (l', q', s') and (l, q, s) are equivalent, *i.e.* they correspond to the same state, if

$$l' = l \quad , \quad q' = q \bmod 2(k+2) \quad , \quad s' = s \bmod 4 \tag{3.7}$$

or

$$l' = k - l \quad , \quad q' = q + k + 2 \quad , \quad s' = s + 2 \tag{3.8}$$

The exact conformal dimension and charge of the highest weight state in the representation (l, q, s) are obtained from equations (3.5) and (3.6) using the identifications above to bring (l, q, s) to the *standard range* given by

$$l = 0, 1, ..., k \quad ; \quad |q - s| \leq l \quad ; \quad l + q + s = 0 \bmod 2 \tag{3.9}$$

and $|s|$ is the minimum value among those in (3.7) and (3.8).

The partition function of the minimal models on the torus can be written in terms of the characters of the irreducible representations as

$$\mathcal{Z}_T^{(m.m.)}(\tau) = \sum_{(l,q,s),(\bar{l},\bar{q},\bar{s})} \mathcal{N}_{(l,q,s),(\bar{l},\bar{q},\bar{s})} \chi_{(l,q,s)}(\tau,0) \chi^*_{(\bar{l},\bar{q},\bar{s})}(\bar{\tau},0) \tag{3.10}$$

where the coefficients $\mathcal{N}_{(l,q,s),(\bar{l},\bar{q},\bar{s})}$ are non negative integer numbers which count the number of times the irreducible representation $(l,q,s)\otimes(\bar{l},\bar{q},\bar{s})$ is contained in $\mathcal{H}$. The existence of a unique ground state requires $\mathcal{N}_{(0,0,0),(0,0,0)} = 1$. The characters in the sector $\mathcal{H}_{(l,q,s)}$ are given by

$$\chi_{(l,q,s)}(\tau,z) = \mathrm{Tr}_{\mathcal{H}_{(l,q,s)}} \left(e^{2\pi i \tau (L_0 - \frac{c}{24})} e^{2\pi i z J_0} \right) \tag{3.11}$$

and it proves useful to define the combination

$$\chi_{l,q}(\tau,z) = \chi_{(l,q,s)}(\tau,z) + \chi_{(l,q,s+2)}(\tau,z) \tag{3.12}$$

Characters of Gepner model are obtained by tensoring the space time and the r internal minimal models characters with the constraint on total $U(1)$ charge to be odd, in order to ensure one supersymmetry. Namely,

$$Q_{tot} = Q_\nu + \sum_{j=1}^{r} Q_{l_j,q_j,s_j} \in 2\mathbb{Z}+1 \tag{3.13}$$

where $\nu = 1,-1,0,2$ refers to spinor, conjugate spinor, scalar and vector representations, respectively. Thus, by defining

$$\chi_{\vec{\alpha}}(\tau,z) = \{[\chi_\nu(\tau,z)]^d \chi_{\alpha_1}(\tau,z)\chi_{\alpha_2}(\tau,z)\ldots\chi_{\alpha_r}(\tau,z)\} \tag{3.14}$$

with

$$\vec{\alpha} = (\alpha_0,\alpha_1,\ldots,\alpha_r) \qquad \alpha_j = (l_j,q_j) \tag{3.15}$$

where $[\chi_\nu(\tau,z)]^d$ is the D dimensional spacetime character with $d = \frac{(D-2)}{2}$.

A *supersymmetric* character, given by

$$\chi^{susy}_{\vec{\alpha}}(\tau,z) = \sum_{n=0}^{2m-1} (-1)^n \chi_{\vec{\alpha}^{(n)}}(\tau,z) = \tag{3.16}$$

$$= \frac{1}{2m} \sum_{n,p \bmod 2m} (-1)^{n+p} e^{2\pi i (n^2 \frac{c}{24}\tau + n\frac{c}{6}z)} \left[\chi_0(\tau, z+\frac{n}{2}\tau+\frac{p}{2})\right]^d \prod_{i=1}^{r} \chi_{l_i,q_i}(\tau, z+\frac{n}{2}\tau+\frac{p}{2})$$

can be built ($c=12$ here). NS or R sectors are obtained when summing over even or odd n respectively, whereas periodic $(+)$ or antiperiodic $(-)$ characters arise when

summing over even or odd p, respectively. Odd charge condition 3.13 is ensured by the sum over p. The *susy* character does transform as the non-susy one and, therefore, full modular invariant partition function is obtained by just coupling the right and left sectors as

$$\mathcal{Z}_T(\tau,\bar{\tau}) = \sum_{\vec{\alpha};\vec{\bar{\alpha}}} \mathcal{N}_{\vec{\alpha};\vec{\bar{\alpha}}} \chi^{susy}_{\vec{\alpha}}(\tau,0) \chi^{susy\ *}_{\vec{\bar{\alpha}}}(\bar{\tau},0) \tag{3.17}$$

$\mathcal{N}_{\vec{\alpha},\vec{\bar{\alpha}}}$ are positive integer coefficients obtained from the product $\prod_{i=1}^{r} \mathcal{N}_{\alpha_i;\bar{\alpha}_i}$ of the individual minimal models. An integration over τ, with the appropriate measure, must then be performed.

Closed sector is obtained by keeping Ω invariant states while open sector can then be easily written down, as linear combinations of these explicitly supersymmetric characters.

3.1 Phase moddings

Gepner models posses a phase symmetry group $G = \otimes_a Z_{M_a}$, associated to phase transformations of primary fields

$$\Phi_{l,q,s} \rightarrow e^{-2i\pi\gamma\frac{q}{m}} \Phi_{l,q,s} \tag{3.18}$$

with $\gamma \in Z$ in each block. Thus, the full phase transformation can be encoded into an r dimensional vector

$$\vec{\Gamma}^a = ({\gamma_1}^a, {\gamma_2}^a, \ldots, {\gamma_r}^a) \tag{3.19}$$

where M_a is the least integer such that

$$M_a {\gamma_i}^a = 0 \qquad \mathrm{mod}\ \ (k_i+2) \tag{3.20}$$

and a labels one of the different, inequivalent, phase transformations. Moddings by such symmetries can be easily implemented [29] by replacing character $\chi_{\vec{l},\vec{q}}(\tau) \rightarrow \chi^G_{\vec{l},\vec{q}}(\tau)$ in 3.17 where the projected character reads

$$\chi^G_{\vec{l},\vec{q}}(\tau) = \frac{1}{M} \sum_{x,y} \chi^G_{\vec{l},\vec{q}}(\tau,x,y) \tag{3.21}$$

We have defined the character in sector (x,y) as

$$\chi^G_{\vec{l},\vec{q}}(\tau,x,y) = e^{-2i\pi x \frac{\gamma_i}{m}(q_i+\gamma_i y)} \chi_{\vec{l},\vec{q}+2\vec{\gamma}y}(\tau) \tag{3.22}$$

where $\vec{l}$, $\vec{q}$ are r-component vectors with entries l_i, q_i respectively. The projection conditions on each twisted closed sector, $y = 0,\ldots,M_a-1$, are

$$\sum_{i=1}^{r} \frac{1}{m_i} \gamma_i^a (q_i + y\gamma_i^a) \in \mathbb{Z} \tag{3.23}$$

while supersymmetry imposes the further constraint ${\gamma_i}^a$,

$$\sum_{i=1}^{r} \frac{1}{m_i} \gamma_i^a \in \mathbb{Z} \tag{3.24}$$

(This is the usual $2\beta_0 \cdot \Gamma \in \mathbb{Z}$ condition of [26]) ensuring integer total charge). A key point in the construction of the projected characters is that they transform as the original ones under modular transformations. Namely,

$$S \ : \ \rightarrow \chi^G_{\vec{\alpha}}(-\frac{1}{\tau}, x, y) = (-i\tau)^{-1} \sum_{\vec{\beta}} S_{\vec{\alpha},\vec{\beta}} \chi^G_{\vec{\beta}}(\tau, -y, x) \tag{3.25}$$

$$T \ : \ \rightarrow \chi^G_{\vec{\alpha}}(\tau + 1, x, y) = e^{i\pi(\Delta_\alpha - \frac{Q_\alpha}{2} - \frac{c}{24})} \chi^G_{\vec{\alpha}}(\tau, x + y, x) \tag{3.26}$$

where $Q_\alpha = -\sum \frac{q_i}{m_i}$, $\Delta_\alpha = \sum \Delta_i$. Notice that same steps can be repeated for right moving sector with characters now depending on $(\bar{x}, \bar{y})$. Moreover, different moddings on right and left moving sectors could be performed.

Interestingly enough, it is possible to think in terms of a kind of hybrid compactification where part of the internal sector is built up from a Gepner model while the rest corresponds to a toroidal like compactification. More specifically, let us assume that we start with a $c = 3n$ $N = 1$ Gepner model in $d = 10 - 2n$ dimensions and that we further compactify $2(3 - n)$ coordinates on a torus in order to obtain a four dimensional model (with extended supersymmetry). A massless, lets say left sector state, would read

$$|r_0, r_1, \ldots, r_{3-n}, (l_i, q_i, s_i)_{i=1,\ldots,r}\rangle \tag{3.27}$$

where r_i $i = 0, \ldots, 3 - n$ are $SO(2(3 - n))$ weight vectors and, a generalized, GSO projection requires

$$\sum_{i=0}^{3-n} r_i - \sum_{j=1}^{r} \frac{1}{2} s_j - \sum_{j=1}^{r} \frac{q_j}{m_j} \quad \in 2\mathbb{Z} + 1 \tag{3.28}$$

If the toroidal sector has a symmetry $\mathbf{Z_N}$ (generically $\mathbf{Z_N} \times \mathbf{Z_M}$), the full internal sector will be invariant under the symmetry group $G = \mathbf{Z_N} \otimes_{\mathbf{a}} \mathbf{Z_{M_a}}$ with $a = 1, \ldots, r$. Orbifolding by such a symmetry, the internal space would look like $(\textit{Gepner model})^{c=3n} \times \mathbf{T^{2(3-n)}}/\mathbf{G}$, for $n = 1, 2, 3$ where orbifold action is encoded in phase moddings vectors Γ of 3.19 of the Gepner sector and eigenvalue vector v 3.3 for the $\mathbf{Z_N}$ orbifold action on the internal tori. Both actions must be performed simultaneously, in a compatible, modular invariant form and will lead to a reduction of supersymmetry. Namely, consider left fermion $Q_{\frac{1}{2}}$ associated to Susy generators. They are of the form shown in 3.27 with $r_0 = 1/2$. If θ realizes a G twist then

$$\theta Q^A_{\mathbf{\frac{1}{2}}} \theta^{-1} = e^{2\pi i (v_i r_i - \frac{\gamma_i q_i}{m_i})} Q^A_{\mathbf{\frac{1}{2}}}. \tag{3.29}$$

Therefore, for supersymmetry to be preserved

$$v_i r_i - \frac{\gamma_i q_i}{m_i} \quad \in \mathbb{Z} \tag{3.30}$$

In particular, a $\mathbf{Z_M}$ subgroup of G, whose action is encoded in twist eigenvalues (Γ, v), can be chosen in order to keep just $N = 1$ supersymmetries.

Thus, the (x, y) sector of a Γ modded Gepner model is accompanied by a (θ^x, θ^y) action on the internal torus. Calling $\chi_{\alpha,x,y}(\tau)$ the partition contribution to such twisted (x, y) sector (and similarly for $(\bar{x}, \bar{y})$) the partition function will formally read

$$Z_T = \frac{1}{M} \sum_{\alpha,\beta,x,y,\bar{x},\bar{y}} \tilde{\chi}(x, y, \bar{x}, \bar{y}) N_{\alpha\beta} e^{-2\pi i \frac{\Gamma x}{m}(q+y\Gamma)} \chi_{\alpha,x,y}(q) e^{-2\pi i \frac{\Gamma \bar{x}}{m}(q-\bar{y}\Gamma)} \chi_{\beta,\bar{x},-\bar{y}}(\bar{q}) \tag{3.31}$$

where $\tilde{\chi}(x, y, \bar{x}, \bar{y})$ is the usual fixed point multiplicity (see [39]) and

$$\chi_{\alpha,x,y}(\tau) = \left\{ \left[\frac{\theta \left[{0 \atop 0} \right]}{\eta^3} \right]^2 \prod_{i=1}^{3-n} e^{i\pi y v_i} \frac{\theta \left[{y v_i \atop x v_i} \right]}{\theta \left[{\frac{1}{2} + y v_i \atop \frac{1}{2} + x v_i} \right]} \chi_{\alpha+2\Gamma y} \right\}_{susy} \tag{3.32}$$

are supersymmetric characters $\chi_{\alpha,x,y}$ for left movers (or right). The first terms in a $q, \bar{q}$ expansion read,($q = e^{-2\pi t}$)

$$\begin{aligned} Z(\tau) \;\; = \;\; & \frac{1}{M} \sum N_{\alpha\beta} \tilde{\chi}(x, y, \bar{x}, \bar{y}) \\ & e^{2\pi i (r+yv)xv} e^{-2\pi i \frac{\Gamma x}{m}(q+y\Gamma)} q^{\frac{1}{2}(r+yv)^2 + E_0(y) + \sum h_{\alpha,y} - \frac{1}{2}} (1 + ...) \\ & e^{-2\pi i (\tilde{r}+\bar{y}v)\bar{x}v} e^{-2\pi i \frac{\Gamma \bar{x}}{m}(\bar{q}-\bar{y}\Gamma)} \bar{q}^{\frac{1}{2}(\tilde{r}+\bar{y}v)^2 + E_0(\bar{y}) + \sum \bar{h}_{\beta,\bar{y}} - \frac{1}{2}} (1 + ...) \end{aligned}$$

where $E_0(y) = \sum_i \frac{1}{2}|yv_i|(1 - |yv_i|)$ is the zero point energy and $h_{\alpha,y}$ is the conformal weight of the primary fields contained in $\chi_{\alpha,x,y}$. Here $r, \tilde{r}$ are $SO(2n+2)$ weight vectors. Thus, for a physical state twisted by y, encoded in the numbers $(r, \alpha)_y \equiv (r, l_i, q_i, s_i)_y$ we can read, for instance, the conditions to have massless state [3]

$$\frac{1}{2}(r + yv)^2 + E_0(y) + \sum h_{\alpha,y} - \frac{1}{2} = \frac{1}{2}(\tilde{r} + \bar{y}v)^2 + E_0(\bar{y}) + \sum \bar{h}_{\alpha,\bar{y}} - \frac{1}{2} = 0 \tag{3.33}$$

with the projector onto invariant states given by

$$D(y, \bar{y}) = \frac{1}{M} \sum_{x,\bar{x}} \tilde{\chi}(x, y, \bar{x}, \bar{y}) e^{2\pi i (r+yv)xv} e^{-2\pi i \frac{\Gamma x}{m}(q+y\Gamma)} e^{-2\pi i (\tilde{r}+\bar{y}v)\bar{x}v} e^{2\pi i \frac{\Gamma \bar{x}}{m}(\bar{q}+\bar{y}\Gamma)} \tag{3.34}$$

Let us return to the pure Gepner case with the aim to deduce an expression for the Klein bottle amplitude. A particular interesting case arises when no modding is performed

[3]For massive states Gepner descendants and oscillators must be included.

on the right sector ($\bar{\Gamma} = 0$) in the *diagonal* invariant partition function. In this case the partition function reads

$$Z_T = \sum_\alpha \left(\frac{1}{M} \sum_{x,y} e^{-2\pi i \frac{\Gamma}{m} x(q+\Gamma y)} \chi_{\alpha+2\Gamma y} \right) \chi^*_\alpha \tag{3.35}$$

which, after summing over x, can be rewritten in the following way

$$Z_T = \sum_\alpha \left(\sum_y \delta \left(\frac{\Gamma}{m}(q+\Gamma y) \right) \chi_{\alpha+2\Gamma y} \right) \chi^*_\alpha . \tag{3.36}$$

Modular invariance can be easily checked from 3.25.

From this latter expression it follows that orbifolding the CFT with respect to phase symmetries induces more general invariant partition function besides the diagonal one.

Klein bottle amplitude is found by keeping identical right and left states. Thus $y = 0 \mod m/2$. In particular, for odd m, only $y = 0$ states are allowed leading to

$$Z_K = \sum_\alpha \delta \left(\frac{\Gamma q}{m} \right) \chi_\alpha(2it) \tag{3.37}$$

We notice that, when m is odd, i.e. the case we are mainly considering in this article, there are no y twisted sectors at all in the direct channel. Z_M action on KB amplitude reduces to a projection onto (Γ) invariant states. Notice, however, that $(x, 0)$ sectors will go into $(0, x)$ twisted sectors in transverse channel and could lead to new contributions to tadpoles. By transforming the Klein bottle amplitude to the transverse channel, ($l = \frac{1}{t}$), previously writing the constraint $\delta\left(\frac{\Gamma q}{m}\right)$ as $\frac{1}{M}\sum e^{-2i\pi\Gamma x \frac{q}{m}}$ we obtain

$$\tilde{Z}_K = \frac{1}{M} \sum_x \sum_{\alpha\beta} 2^D K_\alpha S_{\alpha\beta} \chi_{\beta+2\Gamma x}(il) \tag{3.38}$$

As mentioned, generically, massless RR fields will be present such that $\tilde{Z}_K(il)$ will lead to undesired tadpole divergencies when integrated over l. Therefore D-branes amplitudes must be included in order to cancel such divergences. Similar considerations apply to the hybrid model with the additional complication that, in the present case, characters also depend on both x and y. For odd M, which is the case that we are mainly studying, the Klein bottle amplitude reads

$$Z_K(2it) = \left\{ \frac{1}{M} \sum_{x,\alpha} \left[\frac{\theta \left[{0 \atop 0} \right]}{\eta^3} \right]^2 \prod_{i=1}^{3-n} \frac{\theta \left[{0 \atop xv_i} \right]}{\eta} \frac{-2 \sin \pi x v_i \eta}{\theta \left[{\frac{1}{2} \atop \frac{1}{2}+xv_i} \right]} K_\alpha e^{-2i\pi\Gamma x \frac{q}{m}} \chi_\alpha \right\}_{susy} \tag{3.39}$$

which results from plugging the known partition function for orbifolds and Gepner models.

4 Open sector

Open string states formally read

$$|\Phi_k; i, j\rangle \lambda^k_{ji} \tag{4.1}$$

where λ^k encodes the gauge group representation in which the state Φ_k transforms. For instance, if the state Φ_0 corresponds to gauge bosons, λ^0 represents gauge group G generators [4].

It proves useful [16] to write down Chan Paton matrices in a Cartan-Weyl basis where generators organize into charged generators $\lambda_a = E_a$, $a = 1, \cdots, \dim G$, and Cartan generators $\lambda_I = H_I$, $I = 1, \cdots, \mathrm{rank}\, G$.

In such basis, information about gauge group and matter representations can be encoded into the corresponding root and weight vectors. The allowed λ^k are determined by consistency of the full open string theory, ensuring factorization, tadpole cancellation and classical gauge groups with two indices representations.

Let us assume that such Chan-Paton factors have already been determined and that we further act on string states with a generator θ of a Z_M symmetry group. Such action which manifests as a phase δ_k on world sheet field Φ_k should, in principle, be accompanied by corresponding representation of group action such that

$$\begin{aligned} \hat{\theta}|\Phi_k; i, j\rangle \lambda_{ji} &= \gamma_{ii'}|\hat{\theta}\Phi_k; i', j'\rangle \gamma_{j'j}\lambda_{ji} \\ &= e^{2\pi i\delta_k}(\gamma^{-1}\lambda\gamma)_{j'i'}|\Phi_k; i', j'\rangle \end{aligned}$$

Therefore, invariance under such action requires

$$e^{2\pi i\delta_k}\gamma^{-1}\lambda^k\gamma = \lambda^k \tag{4.2}$$

For odd M and the partition function given in 3.37 we know that $\mathbf{Z_M}$ twist reduces to a projection with no y twisted sector. Thus, for a generic hybrid case where the internal sector of the form $(\textit{Gepner model})^{c=3n} \times \mathbf{T^{2(3-n)}}/\mathbf{Z_M}$, for $(n = 1, 2, 3)$, only open string (v, Γ) invariant states will remain in the spectrum with Chan-Paton factors satisfying the above equation with

$$\delta_k = (v.r - \frac{\Gamma.q}{m}) \tag{4.3}$$

By following the same steps as in Ref. [16], we can represent Z_M Chan-Paton twist in terms of Cartan generators as $\gamma = e^{2\pi i V H}$ where V is a "shift" eigenvalues vector of the generic form

$$V = \frac{1}{M}(0^{N_0}, 1^{N_1}, \ldots, (M-1)^{N_{M-1}}) \tag{4.4}$$

[4] Which generically will be a product of unitary, orthogonal and symplectic groups.

(ensuring $\gamma^M = 1$) and Cartan generators are represented by 2×2 σ_3 submatrices.

In this basis, projection equation 4.2 reduces to the simple condition

$$\rho_k V = \delta_k \tag{4.5}$$

where ρ_k is the weight vector associated to the corresponding λ^k representation.

Similarly we can represent Ω Chan-Paton action in terms of a unitary matrix γ_Ω. More generally, the action of Ωg, $g \in Z_M$, is given by

$$\Omega g : |\Psi, ab\rangle \to (\gamma_{\Omega g,p})_{aa'} |\Omega g \Psi, b'a'\rangle (\gamma_{\Omega g,q})^{-1}_{b'b} \tag{4.6}$$

Consistency with the orientifold group multiplication law implies several constraints on the γ matrices like

$$\gamma_{\Omega g,p} = \gamma_{g,p} \gamma_{\Omega,p} \tag{4.7}$$

or from $(\Omega\theta^x)^2 = \theta^{2x}$,

$$\gamma_{\Omega x,p} = \pm \gamma_{2x,p} \gamma^T_{\Omega x,p}. \tag{4.8}$$

Cancellation of tadpoles imposes further conditions on the γ matrices.

To summarize this section, notice that had we managed to find a consistent model with known Chan Paton factor λ^k, for instance from consistency restrictions on the boundary states and the direct channel amplitudes, phase modded models can then be easily constructed. Open string states are just an invariant subset of the original ones. Recall that, even if the initial group were non-chiral , as it is the case if we started from a diagonal (k odd) invariant (leading to $SO(n)$ or $Sp(n)$ gauge groups), the projection condition on gauge bosons $\rho_k V = 0$ could lead to unitary groups. This works in much the similar way as orbifold invariant states are selected from $SO(32)$ D9-brane Chan-Paton factors in Type I, odd orbifold, compactifications. Certainly, not any projection will be allowed since tadpole cancellation conditions must still be satisfied.

Interestingly enough, for the k even case, $y = 0 \mod m/2$ twisted sector will appear. Thus, new states, absent in the starting theory, will be generically present. This signals the presence of new type of branes with open strings stretching between them as, for instance, D5-branes appear in Type I, even orbifold, compactifications. Extra tadpole cancellation equations will be associated to such states.

4.1 An index formula for Gepner Models

A simple index formula can be explicitly written down for Gepner models. This can be achieved by adapting to Gepner case the expressions derived in [40] for Witten index

$\mathrm{Tr}\,(-1)^F$ in generic conformal field theories, for bifundamental representations. Such an expression could help, in particular, to determine modular invariants and/or phase moddings Γ that could lead to chiral matter content.

We find, for Gepner models (see partition function in 2.8) ,

$$I_{\alpha\beta} = \#(n_\alpha, \bar{n}_\beta) - \#(\bar{n}_\alpha, n_\beta) = \sum_\gamma i e^{i\frac{\pi}{2}Q_\gamma} C^\gamma_{\alpha\beta} \tag{4.9}$$

where $Q_\gamma = -\sum_{i=1}^r \frac{(q_\gamma)_i}{m_i}$.

Notice, for example, that for a diagonal modular invariant, chirality vanishes. In fact, such couplings verify

$$\begin{aligned} C^\gamma_{\alpha\beta} &= C^{\gamma^*}_{\alpha\beta} && (4.10)\\ Q_\gamma &= -Q_{\gamma^*} && (4.11) \end{aligned}$$

where γ^* denotes the vector with components $(\vec{l}_\gamma, -\vec{q}_\gamma)$.

If Gepner model is further modded by discrete symmetries, a projector onto invariant states in the trace $\frac{1}{M}\sum \theta^x$ must be included.

Bifundamental chirality now reads

$$I_{\alpha,j;\beta,i} = \#(n_{\alpha,j}, \bar{n}_{\beta,i}) - \#(\bar{n}_{\alpha,j} n_{\beta,i}) = \sum_\gamma i e^{i\frac{\pi}{2}Q_\gamma} C^\gamma_{\alpha\beta} \delta_{\frac{\Gamma q_\gamma}{m} + V^\alpha_j - V^\beta_i, 0} \tag{4.12}$$

where V_j is given by (4.4). Thus we see that this index is not necessarily vanishing, due to the presence of the δ function.

5 Tadpole cancellation

We proceed to write down the amplitudes in the direct and the transverse channel in order to study factorization and tadpole cancellation. For the sake of simplicity, we first consider the case of (k odd) pure Gepner model and then we generalize to the hybrid case. The cylinder amplitude is given by

$$Z_C(it) = \frac{1}{M}\sum_{x=0}^{M-1}\sum_{\gamma\alpha\beta} C^\gamma_{\alpha\beta}\,\mathrm{tr}\,\gamma_{\alpha,x}\,\mathrm{tr}\,\gamma_{\beta,x} e^{-2i\pi\Gamma\frac{q}{m}x}\chi_\gamma(it) \tag{5.1}$$

which is nothing but the generalization of (2.8) when a projection operator $\frac{1}{M}\sum\theta^x$ is included in the trace. The transverse channel representation of this amplitude reads

$$\tilde{Z}_C(il) = \frac{1}{M}\sum_{x=0}^{M-1}\sum_{\alpha\beta\gamma\delta} C^\gamma_{\alpha\beta} S_{\gamma\delta}\,\mathrm{tr}\,\gamma_{\alpha,x}\,\mathrm{tr}\,\gamma_{\beta,x}\chi_{\delta+2\Gamma x}(il) \tag{5.2}$$

where we have used (3.25). Notice that for each fixed x, (2.11) is verified with $n_a \to \mathrm{tr}\,\gamma_{a,x}$. Namely,

$$C^{\gamma}_{\alpha\beta} S_{\gamma\delta} \mathrm{tr}\,\gamma_{\alpha,x} \mathrm{tr}\,\gamma_{\beta,x} = (D^{\beta}_{\alpha} \mathrm{tr}\,\gamma_{\alpha,x})^2 \tag{5.3}$$

indicating that the transverse amplitude can again be written as a square

$$\tilde{Z}_C(il) = \frac{1}{M} \sum_{x=0}^{M-1} \sum_{\alpha\beta} (D^{\beta}_{\alpha} \mathrm{tr}\,\gamma_{\alpha,x})^2 \chi_{\beta+2\Gamma x}(il) \tag{5.4}$$

Finally, the Möbius strip also contributes at one loop in the open string sector in the following way

$$Z_M(it+1/2) = \frac{1}{M} \sum_{x=0}^{M-1} \sum_{\gamma\alpha} M^{\gamma}_{\alpha} \mathrm{tr}\,(\gamma^{-1}_{\Omega x,\alpha} \gamma^{T}_{\Omega x,\alpha}) e^{-2i\pi\Gamma\frac{q}{m}x} \hat{\chi}_{\gamma}(it+1/2) \tag{5.5}$$

where again we introduce the real *hatted* characters. Thus, transverse Möbius strip amplitude reads

$$\tilde{Z}_M(il+1/2) = \frac{1}{M} \sum_{x=0}^{M-1} \sum_{\gamma\alpha} 2^{\frac{D}{2}} \tilde{M}^{\gamma}_{\alpha} \mathrm{tr}\,(\gamma^{-1}_{\Omega x,\alpha} \gamma^{T}_{\Omega x,\alpha}) \hat{\chi}_{\gamma+4\Gamma x}(il+1/2). \tag{5.6}$$

where $\tilde{M}^{\delta}_{\alpha} = P_{\delta\gamma} M^{\gamma}_{\alpha}$.

Collecting contributions from Klein bottle, cylinder and Möbius strip in the transverse channel we have

$$\begin{aligned} \frac{1}{M} \sum_{x=0}^{M-1} \sum_{\alpha} \{ \quad & (O^{\alpha})^2 \chi_{\alpha}(il) + (D^{\alpha}_{\beta} \mathrm{tr}\,\gamma_{\beta,x})^2 \chi_{\alpha}(il) + \\ & 2 \times 2^{\frac{D}{2}} \tilde{M}^{\alpha}_{\beta} \mathrm{tr}\,(\gamma^{-1}_{\Omega x,\beta} \gamma^{T}_{\Omega x,\beta}) \hat{\chi}_{\alpha}(il+\frac{1}{2}) \} \end{aligned} \tag{5.7}$$

Notice that despite there are no Γ twisted sectors in the direct channel, projections lead to x twisted sectors in the transverse one.

Factorization (2.6) for $l \to \infty$, amounts to

$$(D^{\alpha}_{\beta} \mathrm{tr}\,\gamma_{\beta,2x})^2 + 2 \times 2^{\frac{D}{2}} \tilde{M}^{\alpha}_{\beta} \mathrm{tr}\,(\gamma^{-1}_{\Omega x,\beta} \gamma^{T}_{\Omega x,\beta}) + (O^{\alpha})^2 = \text{perfect square} \tag{5.8}$$

Namely, $2^{\frac{D}{2}} \tilde{M}_{\alpha} = D_{\alpha} O_{\alpha}$ and

$$\mathrm{tr}\,(\gamma^{-1}_{\Omega x,\alpha} \gamma^{T}_{\Omega x,\alpha}) = \pm \mathrm{tr}\,\gamma_{2x,\alpha} \tag{5.9}$$

the same type of condition as found in orbifold compactifications [41]. Interestingly enough, we see that, with this condition, factorized amplitudes in the modded theory

can be immediately written down from the unprojected theory by just performing the replacement

$$n_\alpha \to \mathrm{tr}\,\gamma_{\alpha,x} \tag{5.10}$$

Moreover, zero charge condition for RR massless fields (2.7) is easily found by requiring

$$D_a^\alpha \mathrm{tr}\,\gamma_{a,x} + O^\alpha = 0 \tag{5.11}$$

for characters $\chi_{\alpha+2\Gamma x}$ containing massless RR states.

In particular, for untwisted $x = 0$ transverse sector the original conditions are recovered since, $\mathrm{tr}\,\gamma_{a,0} = n_a$. As mentioned, when diagonal invariants are considered, the number of tadpole conditions is smaller than with other invariants, and therefore easier to solve.

Nevertheless after modding out phase symmetries this number rises and it corresponds to the number of massless twisted transverse characters.

Tadpole condition can be generalized for hybrid models $T^{2(3-n)} \times$ *Gepner* and Z_N modding, odd N, in the following way

$$D_\beta(\mathrm{tr}\,\gamma_{\alpha,x}) + 2O_\beta \prod_{i=0}^{3-n} 2\cos\pi x v_i = 0 \tag{5.12}$$

which arises from transforming to the transverse channel (3.39) and the open sector amplitudes

$$Z_C(it) = \left\{ \sum_{\alpha\beta\gamma} \left[\frac{\theta\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]}{\eta^3} \right]^2 \prod_{i=1}^{3-n} \frac{\theta\left[\begin{smallmatrix}0\\ xv_i\end{smallmatrix}\right]}{\eta} \frac{-2\sin\pi x v_i \eta}{\theta\left[\begin{smallmatrix}\frac{1}{2}\\ \frac{1}{2}+2xv_i\end{smallmatrix}\right]} C_{\alpha\beta}^\gamma e^{-2i\pi\Gamma x\frac{q}{m}} \chi_\gamma \mathrm{Tr}\,\gamma_{x,\alpha} \mathrm{Tr}\,\gamma_{x,\beta} \right\}_{susy}$$

$$Z_M(it+\frac{1}{2}) = \left\{ \sum_{\alpha a} \left[\frac{\theta\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]}{\eta^3} \right]^2 \prod_{i=1}^{3-n} \frac{\theta\left[\begin{smallmatrix}0\\ xv_i\end{smallmatrix}\right]}{\eta} \frac{-2\sin\pi x v_i \eta}{\theta\left[\begin{smallmatrix}\frac{1}{2}\\ \frac{1}{2}+2xv_i\end{smallmatrix}\right]} M_a^\alpha e^{-2i\pi\Gamma x\frac{q}{m}} \hat{\chi}_\alpha \mathrm{Tr}\,[\gamma_{\Omega x,a}^{-1} \gamma_{\Omega x,a}^T] \right\}_{susy}$$

We should remember that according to the combined action of the orbifold twist and Ω, a sum over quantized momenta or over windings must be included (see [16]). To arrive at the tadpole cancellation conditions, we must take the limit $t \to 0$ in the various traces and next change variable to l appropriately to find the large l behavior of the amplitudes. The final step is to collect all terms with a given volume dependence.

6 Examples

In this section we exhibit some explicit examples of $D = 4$ chiral models by following the general steps discussed above. The situation in which the internal sector is *purely*

Gepner is illustrated by considering phase moddings of 3^5_D quintic. The hybrid situation of a Gepner-orbifold internal sector is exemplified by considering orbifolds of $(1)^6 \times T^2$ and $(1)^3 \times T^4$ internal sector models. The latter is a peculiar model, in some sense, since the Gepner part is, actually, also a (special) torus. Nevertheless, it is useful not only to illustrate the general method but to show how models where rank is reduced are obtained . By closely following Ref [42] we show how antibranes can be included in these constructions.

6.1 $\mathbf{3^5_D/Z_5}$

A consistent open string theory with internal sector given by $\mathbf{3^5}$ Gepner model, with a diagonal invariant, was found in [29] from where we borrow results and notation (see also [30] and [32]). The KB partition function can be written as

$$\mathcal{Z}_K(it) = \frac{1}{2}\frac{1}{5}\left[(\chi_I(it) + \chi_{II}(it))^5\right]^{susy} \tag{6.1}$$

where

$$\begin{aligned} \chi_I &= \chi_{(0,0)} + \chi_{(3,-3)} + \chi_{(3,-1)} + \chi_{(3,1)} + \chi_{(3,3)} \\ \chi_{II} &= \chi_{(2,0)} + \chi_{(2,2)} + \chi_{(1,-1)} + \chi_{(1,1)} + \chi_{(2,-2)} \end{aligned} \tag{6.2}$$

which reads, in the transverse channel

$$\tilde{\mathcal{Z}}_K(il) = \frac{1}{2}2^4\sqrt[4]{5}\left[(\kappa^{\frac{3}{2}}\tilde{\chi}_{(0,0)}(il) + \kappa^{-\frac{3}{2}}\tilde{\chi}_{(2,0)}(il))^5\right]^{susy} \tag{6.3}$$

where $\kappa \equiv \frac{1}{2}(1+\sqrt{5})$. The partition function in the transverse channel can be written in terms of $\tilde{\chi}_{(0,0)}(il)^{1-\gamma_i}\tilde{\chi}^{\gamma_i}_{(2,0)}$ (where the exponents indicate the number of times each factor appears, regardless of order). Each term is encoded in a 5 component vector (one for each theory) $\vec{\gamma}$ taking values 0 or 1 (which corresponds to a state belonging to group I or II in the direct channel, respectively) For instance, by rewriting 6.3 as,

$$\tilde{\mathcal{Z}}_K(il) == \frac{1}{2}\mathcal{O}^2_{\vec{\gamma}}\left[\prod_{i=1}^{5}(\tilde{\chi}_{(0,0)}(il))^{1-\gamma_i}(\tilde{\chi}_{(2,0)}(il))^{\gamma_i}\right]^{susy} \tag{6.4}$$

we find that the only non vanishing coefficients are

$$\mathcal{O}_{\vec{0}} = 2^4 5^{\frac{1}{8}}\kappa^{\frac{15}{2}} \qquad ; \qquad \mathcal{O}_{\vec{1}} = 2^4 5^{\frac{1}{8}}\kappa^{-\frac{15}{2}} \tag{6.5}$$

where $\vec{0} \equiv (0,0,0,0,0)$ and $\vec{1} \equiv (1,1,1,1,1)$.

Similarly $\mathcal{D}_{\vec{\gamma}}$ and $\mathcal{M}_{\vec{\gamma}}$ coefficients are defined for the cylinder and MS amplitudes. Consistency is ensured for [29]

$$\mathcal{D}^2_{\vec{\gamma}} \;=\; \frac{5^{\frac{1}{4}}}{\kappa^{5/2}} \frac{\left(\sum_{\vec{\delta}} \kappa^{(\vec{\gamma}-\vec{\delta})^2}(-1)^{\vec{\gamma}.\vec{\delta}} n^{\vec{\delta}}\right)^2}{\kappa^{(\vec{\gamma})^2}} \tag{6.6}$$

and

$$\tilde{\mathcal{M}}_{\vec{\gamma}} = \tilde{\mathcal{D}}_{\vec{\gamma}}\tilde{\mathcal{O}}_{\vec{\gamma}} = -\sum_{\vec{\delta}} \sqrt[4]{5}(-1)^{(\vec{\gamma})^2}\kappa^{(\vec{\gamma}-\vec{\delta})^2}(-1)^{\vec{\gamma}.\vec{\delta}}\kappa^{\frac{5}{2}-2(\vec{\gamma})^2} n_{\vec{\delta}}. \tag{6.7}$$

Tadpole cancellation equations are thus

$$\mathcal{D}_{\vec{0}} + \mathcal{O}_{\vec{0}} \;=\; 0 \tag{6.8}$$

$$\mathcal{D}_{\vec{1}} + \mathcal{O}_{\vec{1}} \;=\; 0 \tag{6.9}$$

which read,

$$\begin{aligned} N_0 + N_2 + N_3 + 2N_4 + 3N_5 &= 12 \\ N_1 + N_2 + 2N_3 + 3N_4 + 5N_5 &= 20 \end{aligned} \tag{6.10}$$

where $N_i = \sum_{\vec{\gamma}\ /\ |\vec{\gamma}|=i} n_{\vec{\gamma}}$ (e.g., $n_4 = n_{(1,1,1,1,0)} + n_{(1,1,1,0,1)} + n_{(1,1,0,1,1)} + n_{(1,0,1,1,1)} + n_{(0,1,1,1,1)}$). By studying the direct channel expressions the open string spectrum can be found [29]. The gauge group is of the form $\prod_{i=0}^{5} SO(n_i)$ with matter states transforming in antisymmetric, symmetric or bifundamental representations. For instance, with $N_0 = n_{(0,0,0,0,0)}; N_1 = n_{(1,0,0,0,0)}; N_2 = n_{(1,1,0,0,0)}$ (and all other entries vanishing) the following massless spectrum is found

S-T	Internal	mult.	irrep.
v	$(0,0)^5$	1	$SO(N_0)\otimes SO(N_1)\otimes SO(N_2)$
s	$(2,2)\underline{(3,3)(0,0)^3}$	4	$(1,\square\!\square,1)+(1,1,\square\!\square)+(N_0,N_1,1)$
s	$(3,3)(2,2)(0,0)^3$	1	$(1,1,\square\!\square)+(1,N_1,N_2)$
s	$(0,0)(2,2)\underline{(0,0)^2(3,3)}$	3	$(1,1,\square\!\square)+(1,N_1,N_2)$
s	$(1,1)^2\underline{(0,0)^2(3,3)}$	3	$(1,1,\begin{smallmatrix}\square\\\square\end{smallmatrix})+(N_0,1,N_2)+(1,N_1,N_2)$

with

$$N_0 = 12 - N \quad ; \quad N_1 = 20 - N \quad ; \quad N_2 = N \tag{6.11}$$

v, s indicate that such states are vector or chiral scalar superfields, respectively.

Phase symmetries of the 3^5 allow for 124 different independent moddings. Moreover, more than one modding could be simultaneously performed. By embedding such moddings as twists on D-branes chiral models can be obtained. In order to illustrate the procedure let us consider the simple situation when $N = 0$. Thus, our starting point is a

$$\begin{array}{l} SO(12) \otimes SO(20) \\ 4[(1, \square\!\square) + (12, 20)] \end{array} \tag{6.12}$$

where, as can be read from the table, the multiplicity comes from possible permutations in

$$(2,2)(3,3)(0,0)^3 + (2,2)(0,0)(3,3)(0,0)^2 + (2,2)(0,0)^2(3,3)(0,0) + (2,2)(0,0)^3(3,3)$$

We choose to perform the modding $\Gamma = (0, 2, -1, -1, 0)$ and to embed it as the generic Chan-Paton twist

$$V = \frac{1}{5}(0^{n_0}, 1^{n_1}, 2^{n_2}; 0^{m_0}, 1^{m_1}, 2^{m_2}) \tag{6.13}$$

with

$$\begin{aligned} \frac{1}{2}N_0 &= n_0 + n_1 + n_2 = 6 \\ \frac{1}{2}N_1 &= m_0 + m_1 + m_2 = 10 \end{aligned} \tag{6.14}$$

Spectrum is obtained by projecting the above states according to eq.4.2. Here $\delta = \frac{\Gamma.q}{5} = 0, -\frac{1}{5}, \frac{2}{5}, \frac{2}{5}, 0$ for the gauge bosons and the four respective massless matter states while, for instance,

$$\rho_{(Adj,1)} = (\underline{\pm 1, \pm 1, 0, \ldots, 0}; 0 \ldots, 0) \tag{6.15}$$

$$\rho_{(1,\square\!\square)} = (0, \ldots, 0; (\underline{\pm 1, \pm 1, 0, \ldots, 0})) + (0, \ldots, 0; \underline{\pm 2, \ldots, 0}) \tag{6.16}$$

$$\rho_{(12,20)} = (\underline{\pm 1, 0, \ldots, 0}; \underline{\pm 1, 0, \ldots, 0}) \tag{6.17}$$

where the first (second) 6 (10) entries correspond to $SO(12)$ ($SO(20)$) weight vectors. Thus, we see, for instance that the original gauge group breaks to $SO(2n_0) \times U(n_1) \times U(n_2) \times SO(2m_0) \times U(m_1) \times U(m_2)$. Matter states can be easily computed. Spectrum is generically chiral but anomalous for arbitrary values of n's and m's. In fact, strong restrictions are imposed by tadpole cancellation.

As showed above, tadpole cancellation equations for the projected theory can be easily obtained from the unprojected theory (see 5.11.) We just have to replace $n_a \rightarrow$

$\mathrm{tr}\,\gamma_{a,x}$ in the transverse channel expressions for the corresponding x twisted characters. Namely,

$$\begin{array}{lcl} N_0 & \rightarrow & \mathrm{tr}\,\gamma_{0,x} = 2n_0 + 2n_1 \cos\frac{2}{5}\pi x + 2n_2 \cos\frac{4}{5}\pi x \\ N_1 & \rightarrow & \mathrm{tr}\,\gamma_{1,x} = 2m_0 + 2m_1 \cos\frac{2}{5}\pi x + 2m_2 \cos\frac{4}{5}\pi x \end{array} \tag{6.18}$$

Tadpole cancellation equations thus read

$$(D_a^\alpha \mathrm{tr}\,\gamma_{a,x} + O^\alpha)^2 \chi_{\alpha+2\Gamma x}(il) = 0 \qquad (l \rightarrow \infty) \tag{6.19}$$

for all x twisted states such that $\vec{\alpha} + 2\Gamma x$ contains an RR massless state. For the example at hand such states are given in table 1 below

$\vec{\alpha}$					x	$\vec{\alpha} + 2\Gamma x$ (massless)				
(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	0	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	0	(1,-1)	(1,-1)	(1,-1)	(1,-1)	(1,-1)
(0,0)	(2,0)	(2,0)	(2,0)	(0,0)	1	(0,0)	(1,-1)	(2,-2)	(2,-2)	(0,0)
(2,0)	(0,0)	(0,0)	(0,0)	(2,0)	2	(1,-1)	(3,-3)	(0,0)	(0,0)	(1,-1)
(0,0)	(0,0)	(2,0)	(2,0)	(0,0)	3	(0,0)	(3,-3)	(1,-1)	(1,-1)	(0,0)
(2,0)	(2,0)	(0,0)	(0,0)	(2,0)	4	(2,-2)	(1,-1)	(0,0)	(0,0)	(2,-2)

Table 1: Massless twisted states $\vec{\alpha} + 2\Gamma x$

and lead to the equations

$$\begin{array}{rcl} 44 + 20\sqrt{5} & = & \pm(2\ \mathrm{tr}\,\gamma_{0,0} +\ \mathrm{tr}\,\gamma_{1,0} + \sqrt{5}\ \mathrm{tr}\,\gamma_{1,0}) \\ 8 & = & \pm(11\ \mathrm{tr}\,\gamma_{0,0} + 5\sqrt{5}\ \mathrm{tr}\,\gamma_{0,0} - 7\ \mathrm{tr}\,\gamma_{1,0} - 3\sqrt{5} n_1) \\ 12 + 4\sqrt{5} & = & \pm(4\ \mathrm{tr}\,\gamma_{0,2} + 2\sqrt{5}\ \mathrm{tr}\,\gamma_{0,2} + 7\ \mathrm{tr}\,\gamma_{1,2} + 3\sqrt{5}\ \mathrm{tr}\,\gamma_{1,2}) \\ 12 + 4\sqrt{5} & = & \pm(4\ \mathrm{tr}\,\gamma_{0,2} + 2\sqrt{5}\ \mathrm{tr}\,\gamma_{0,2} - 3\ \mathrm{tr}\,\gamma_{1,2} - \sqrt{5}\ \mathrm{tr}\,\gamma_{1,2}) \\ 16 + 8\sqrt{5} & = & \pm(3\ \mathrm{tr}\,\gamma_{0,4} + \sqrt{5}\ \mathrm{tr}\,\gamma_{0,4} + 4\ \mathrm{tr}\,\gamma_{1,4} + 2\sqrt{5}\ \mathrm{tr}\,\gamma_{1,4}) \\ 16 + 8\sqrt{5} & = & \pm(3\ \mathrm{tr}\,\gamma_{0,4} + \sqrt{5}\ \mathrm{tr}\,\gamma_{0,4} -\ \mathrm{tr}\,\gamma_{1,4} - \sqrt{5}\ \mathrm{tr}\,\gamma_{1,4}) \end{array}$$

Sign freedom is due to the fact that both signs lead to square completion. The first two equations are the original, untwisted ones (6.18) fixing the total group ranks. The extra, *x twisted*, equations can be easily checked to be the conditions to be satisfied for the theory to be *anomaly free*. The solution to these tadpole equations is unique in this case (it corresponds to sign selection$\{+,+,-,+,-,-\}$), namely

$$n_0 = 2 \quad n_1 = 4 \quad n_2 = 0$$
$$m_0 = 2 \quad m_1 = 4 \quad m_2 = 4$$

leaving an $SO(4)\times U(4)\times SO(4)\times U(4)\times U(4)$ gauge group with chiral matter content

$$(1,1;4,4,1) + (1,1;1,\bar{4},4) + (1,1;1,1,\overline{10}) + (1,\bar{4};1,1,4) + (1,4;1,\bar{4},1) + (4,1;1,4,1) +$$
$$2[1,1;1,10,1) + (1,1;1,\bar{4},\bar{4}) + (1,1;4,1,4) + (1,\bar{4};1,\bar{4},1) + (4,1;1,1,4) + (1,4;4,1,1)]$$

We thus see that unitary groups with chiral matter can be easily obtained.

A biased search by considering simultaneous moddings should be performed in order to look, for instance, for models closer to the Standard model or some extension of it. We are not attempting this search here.

Nevertheless, we have checked the spectrum for the 124 independent moddings. For the $SO(N_0)\times SO(N_1)\times SO(N_2)$ case of (6.11) we found that sixteen of them lead to inconsistent models where tadpoles can not be cancelled. For some of the allowed moddings more than one solution to tadpole equations exist.

The following 36 moddings lead only to non-chiral models:

$$\Gamma = (0,0,\underline{0,1,-1}), (0,0,\underline{0,2,-2}), \pm(0,1,\underline{-1,2,-2}), \pm(0,2,\underline{1,-1,-2})$$

The other 72 all lead to at least a solution with chiral spectra. Unitary groups with high ranks, up to $U(10)$ are found. However, only groups of up to rank four have chiral spectra (see also [31]). Clearly further projections leading to further breaking might lead to other possibilities.

We have checked, in some examples, that twisted tadpole cancellation does coincide with anomaly cancellation conditions.

6.2 $(\mathbf{1^3} \times \mathbf{T^4})/\mathbf{Z_3}$

An open string theory with internal sector given by $(1)^3$ Gepner model, with a diagonal invariant, was found in [29] . In this case the KB, MS and C partition functions can be written in the transverse channel as

$$\tilde{Z}_K + \tilde{Z}_M + \tilde{Z}_C = 2^8 \tilde{\chi}^{susy}_{(0,0)^3}(il) - 2\times 2^4 \hat{\tilde{\chi}}^{susy}_{(0,0)^3}(il + \frac{1}{2}) + n^2 \tilde{\chi}^{susy}_{(0,0)^3}(il). \qquad (6.20)$$

Since $\chi^{susy}_{(0,0)^3}$ contains massless RR states, tadpole cancellation equation is thus $n - 16 = 0$. By studying the direct channel expressions the open string spectrum can be found. Gauge group is Sp(n) with massive matter states transforming in antisymmetric or symmetric representation. By compactifying once more on a T^4 torus, we obtain a four-dimensional string theory with open sector massless spectrum shown in table 2.

Space-time	Internal	Irrep.
v	$(0,0)(0,0,0)^3$	Sp(16)
s	$(1,0)\ (0,0,0)^3$	$\square\!\square$
s	$(0,1)\ (0,0,0)^3$	$\square\!\square$
s	$(0,0)(1,-1,0)^3$	$\square\!\square$

Table 2: Open sector massless spectrum

These states make up a $Sp(16)$ vector multiplet and 3 chiral superfields transforming in ($\square\!\square$) . Phase symmetries of the $(1)^3 \times T^4$ allow for two different independent moddings

$$\Gamma = (1,-1,0) \quad v = (0, \frac{1}{3}, -\frac{1}{3}) \tag{6.21}$$

$$\Gamma = (1,1,0) \quad v = (0, \frac{1}{3}, \frac{1}{3}) \tag{6.22}$$

though the first one leads to N=2 supersymmetries in space-time and hence to non-chiral models. By embedding the second modding as twist on D-branes chiral models can be obtained. Consequently, we choose to perform the modding (6.22) and to embed it as the generic Chan-Paton twist

$$V = \frac{1}{3}(0^{n_0}, 1^{n_1}) \tag{6.23}$$

with $n_0 + n_1 = 8$. Spectrum is obtained by projecting above states according to eq. (4.5). Here $\delta = \frac{\Gamma q}{m} = 0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ for the gauge bosons and three respective massless states. Thus, from (6.23) and (4.5) we obtain that the original gauge group breaks to $Sp(2n_0) \times U(n_1)$. Matter states can be easily computed and lead to a $3[(2n_0, n_1) + (1, \overline{\square\!\square})]$ chiral representation. Spectrum is chiral but anomalous for arbitrary values of n_0, n_1. However, strong restrictions imposed by tadpole cancellation will ensure anomaly-free spectrum.

As we have shown, above tadpole cancellation equations for the projected theory can be easily obtained from the transverse channel expressions. Tadpole cancellation equations given in (5.12) thus read

$$\mathrm{Tr}\, \gamma_x - 2(2\cos\frac{\pi x}{3})^2 = 0 \tag{6.24}$$

for all x such that $\chi_{\alpha+2\Gamma x}$ contains a RR massless state. Such states are given in table 3 below.

x	$\alpha + 2\Gamma x =$ St $\times$ Gepner
0	$(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2});(0,1,1)(0,1,1)(0,1,1)$ + others
1	$(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2});\underline{(0,-1,-1)(0,-1,-1)(0,1,1)}$
2	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2});\underline{(0,1,1)(0,1,1)(0,-1,-1)}$

Table 3: RR massless states in transverse channel

They lead to the equations

$$n_0 + n_1 = 8 \tag{6.25}$$

$$2n_0 - n_1 = 4. \tag{6.26}$$

The first equation is the original, untwisted one. The extra, twisted, equation is just the condition for the theory to be anomaly free.

The solution to these tadpole equations is unique, namely $n_0 = n_1 = 4$ leaving a $Sp(8) \times U(4)$ gauge group with chiral matter content $3[(8,4) + (1, \overline{\square\square})]$.

Similarly to the 3^5 Gepner model we see that unitary groups with chiral matter can be easily obtained in the hybrid models. Moreover, this example leads to three matter generations. It is interesting to compare this computation with an orbifold like case. For instance, in a type-IIB orientifold on T^6/Z_3 the massless spectrum reads

$$Vector \qquad SO(2n_0) \times U(n_1) \tag{6.27}$$

$$Chiral \qquad 3[(2n_0, n_1) + (1, \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}})] \tag{6.28}$$

and the tadpole cancellation conditions are

$$n_0 + n_1 = 16 \tag{6.29}$$

$$2n_0 - n_1 = -4. \tag{6.30}$$

Thus, compared to our case, we see that they are similar if we make the following replacement: $SO(n) \to Sp(n)$ and $\begin{smallmatrix}\square\\ \square\end{smallmatrix} \to \square\square$. Besides, notice that the Gepner model leads to a rank reduction which can be explained from the presence of a NSNS antisymmetric field.

6.3 Non-supersymmetric standard like models

In this section we consider a non-supersymmetric variation of the $(1)^3 \times T^4$ model discussed in the previous section. We show that by finding a specific form of the

twist matrices, tadpole cancellation might require the presence of parallel branes and antibranes on the torus $T^{2(3-n)}$ sector. In addition, adding Wilson lines, for example one wrapped in the e_1 direction in the first complex plane, will enormously increase the freedom to construct phenomenologically interesting three generations models.

Let us begin with a $(1)^3 \times T^4$ model and add open strings with the following boundary conditions

$$\begin{aligned} n^a \partial_a X^\mu &= 0, \quad p = 0, ..., 7 && (6.31) \\ X^i &\in (1)^3, \quad i = 8, 9 && (6.32) \end{aligned}$$

and

$$\begin{aligned} n^a \partial_a X^\mu &= 0, \quad p = 0, ..., 3 && (6.33) \\ X^i &= 0, \quad i = 4, 5, 6, 7 && (6.34) \\ X^i &\in (1)^3, \quad i = 8, 9 && (6.35) \end{aligned}$$

Both groups of conditions define that open string ends live, respectively, in what we have generically called, DQ-branes and DP-branes. In order to cancel untwisted tadpoles the orientifold action requires the introduction of 16 DQ-branes and a zero net number of DP-branes. However, it is also possible to include DP-branes if an equal number of D$\bar{P}$-antibranes is introduced (wrapping, for instance, the third complex plane). This leads to new (non-supersymmetric) consistent models. This example is closely related to the non-supersymmetric Z^3 orientifold of [43]. Branes and antibranes annihilation can be prevented by considering models with branes and antibranes stuck at different fixed points in T^4. The tadpole cancellation conditions read

$$\mathrm{Tr}\,\gamma_Q + 3(\mathrm{Tr}\,\gamma_{P,L} - \mathrm{Tr}\,\gamma_{\bar{P},L}) = 4 \qquad (6.36)$$

for any of the nine fixed points L in the two first complex planes. Also the number of DP-branes and D$\bar{P}$-antibranes must be the same. Notice that choosing $\mathrm{Tr}\,\gamma_Q \neq 4$ inevitably demands the presence of DP-branes and/or D$\bar{P}$-antibranes at all fixed points. We also include a Wilson line W wrapping along direction e_1 in the first complex plane which modifies the tadpole cancellation equations

$$\mathrm{Tr}\,W^k\gamma_Q + 3(\mathrm{Tr}\,\gamma_{P,L} - \mathrm{Tr}\,\gamma_{\bar{P},L}) = 4 \qquad k = 0, 1, 2 \qquad (6.37)$$

We choose to perform the modding (6.22) and to embed it as the generic Chan-

Paton twists

$$
\begin{aligned}
V_Q &= \frac{1}{3}(0^{N_0}, 0^{N_1}, 1^{N_2}, 1^{N_3}, 1^{N_4}) && (6.38)\\
W &= \frac{1}{3}(0^{N_0}, 1^{N_1}, 0^{N_2}, 1^{N_3}, 2^{N_4}) && (6.39)\\
V_P &= \frac{1}{3}(0^{m_L}, 1^{n_L}) && (6.40)\\
V_{\bar{P}} &= \frac{1}{3}(0^{p_L}, 1^{q_L}) && (6.41)
\end{aligned}
$$

with $N_0 + N_1 + N_2 + N_3 + N_4 = 8$. Spectrum is obtained by projecting states as above, but now it contains new states coming from different open string sectors, namely, QQ, PP, QP, Q$\bar{P}$, etc. For instance, bosonic massless states in the QP sector will be given by

$$
(0, \frac{1}{2}, \frac{1}{2})(0,0,0)^3 \tag{6.42}
$$

due to the fact that there are mixed DN boundary conditions in the $X^p, p = 4,5,6,7$ coordinates. Thus, we see, for example, that the original gauge group in the QQ sector breaks to

$$
Sp(2N_0) \times \prod_{s=1}^{4} U(N_s). \tag{6.43}
$$

Matter states can be easily computed following the steps used in the Z_3 orientifold of [42]. We leave this long computations to the Appendix. We illustrate the results in some interesting examples below. Considering three different actions of the twist on the Chan-Paton factors $\{N_i\}$ we lead to the models in table 4.

Observable Group	Chiral Matter (3 families)
$SU(3) \times SU(2) \times U(1)_Y$	$(3,1)_{\frac{2}{3}} + (\bar{3},2)_{-\frac{1}{6}} + (3,1)_{-\frac{1}{3}}$ $(1,1)_{-1} + (1,2)_{+\frac{1}{2}}$
$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$	$(3,2,1)_{-\frac{1}{3}} + (\bar{3},1,2)_{+\frac{1}{3}} + (1,2,2)_0$ $(1,2,1)_{-1} + (1,1,2)_{+1}$
$SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$	$(4,2,1)_{-\frac{1}{2}} + (\bar{4},1,2)_{+\frac{1}{2}} + (1,2,2)_0$ $2(1,1,2)_{-1} + 2(1,2,1)_{+1}$

Table 4: Standard-like models

We see that Standard like model, Left-Right symmetric models or Pati-Salam models with three generations can be easily constructed. Interestingly enough, the allowed group, rank 8, is fulfilled by the Pati-Salam group in the QQ sector.

In addition to above content non-chiral matter transforming under observable gauge group representations and matter in some hidden gauge sector generically appear. Following the general procedure of [42] we have identified one anomaly free combination of the U(1) factors which works as hypercharge.

Recall that while open sector is non supersymmetric, closed sector has $N = 1$ susy, ensured by 3.30, and no closed tachyon is present.

6.4 $(Gepner\ model)^{c=6} \times \mathbf{T^2}$

Cases in which the internal sector is an orbifold of $c = 6$ Gepner models times a two torus can be also considered. A very simple, toy example, is provided by starting with $(1)^6_D$ model of [29], and then modding out phase symmetries encoded in $v = (0, \frac{2}{3})$ and $\Gamma = (-\frac{1}{3}, -\frac{1}{3}, 0, 0, 0, 0)$. $U(4)$ gauge group with matter in $\mathbf{6}$ or $\bar{\mathbf{6}}$ is obtained. At first sight, DP branes could be introduced, as we did in previous $(1)^3 \times T^4$ example, and look for higher rank non-susy extensions by addition of antibranes. However, we find that PP and QQ sectors decouple in this case, and hence do not lead new massless states. Further investigation reveals that configurations with DP-branes can exist whenever the Gepner model part contains states with level k even. Interesting non-susy models could be obtained in such cases. We will not analyse them here.

7 Summary and outlook

In this paper we have discussed the construction of Type IIB orientifold models where the internal "compactified" sector is obtained as a discrete $\mathbf{Z_N}$ like symmetry projection of $(Gepner\ model)^{c=3n} \times \mathbf{T^{2(3-n)}}$ space. The projection is realized as the combined action of a phase symmetry of Gepner sector and rotation of torus lattice. Such symmetry action is embedded as a twist on Chan-Paton factors in open sector and leads to restrictions on them. By using a Cartan-Weyl basis such restrictions become projections on weight vectors which are very easy to handle. Generically unitary groups with chiral matter representations are found. We have presented an index formula which allows for a further control on the number of chiral representations.

One interesting outcome of this hybrid construction is that, presence of parallel branes on the torus $\times\mathbf{T^{2(3-n)}}$ sector, should allow for lowering the string scale.

A possible strategy, used in our paper, is to start with simple modular invariants for Gepner models, for instance a diagonal one, leading to a small number of tadpole equations. Once a consistent solution is found, we further project by the corresponding

symmetry in order to obtain unitary groups, chiral matter etc. This works in a rather similar way as orbifolds projections of $SO(32)$ group of Type I string theory. Tadpole equations are easy to formulate once the starting theory is known. An alternative, somewhat opposite, application of phase moddings is to use them to reduce the number of tadpole equations to solve [31].

In this article we have concentrated in some simple examples in order to illustrate the method rather to attempt a systematic search for phenomenologically interesting models. An important advantage of the procedure is that several results can be obtained analytically.

Non supersymmetric, tachyon free, open string models were constructed by introducing antibranes, following [42, 43]. Tadpole equations prevent them to annihilate. However, the issue of moduli stabilization remains as an open problem (see for instance [44]). We have shown, in the models considered, that anomaly cancellation is ensured by tadpole cancellation.

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8 Appendix

8.1 Non-supersymmetric $(1)^3 \times T^4$ spectrum

Let us considerer the following twist matrices

$$\begin{aligned}
\tilde{\gamma}_Q &= diag(1_{N_0}, 1_{N_1}, \alpha 1_{N_2}, \alpha 1_{N_3}, \alpha 1_{N_4}) &(8.1)\\
\tilde{\gamma}_{P,i,a} &= diag(1_{2m_a^i}, \alpha 1_{n_a^i}, \alpha^2 1_{n_a^i}) &(8.2)\\
\tilde{\gamma}_{\bar{P},j,a} &= diag(1_{2p_a^j}, \alpha 1_{q_a^j}, \alpha^2 1_{q_a^j}) &(8.3)
\end{aligned}$$

with $\alpha = e^{2\pi i/3}$ and $N_0 + N_1 + N_2 + N_3 + N_4 = 8$ and where nine orbifold fixed points were labelled as $(a,i), a,i = 0,1,2$. Since the number of branes and antibranes must be the same we must have

$$\sum_{ia}(n_a^i + m_a^i) = \sum_{bj}(p_b^j + q_b^j) \tag{8.4}$$

We also include a Wilson line $W = (\tilde{W}, \tilde{W}^*)$ wrapping along direction e_1 in the first complex plane

$$W = diag(1_{N_0}, \alpha 1_{N_1}, 1_{N_2}, \alpha 1_{N_3}, \alpha^2 1_{N_4}) \tag{8.5}$$

with $\alpha = e^{2\pi i/3}$.

The tadpole cancellation conditions when a Wilson line is turned on read

$$\mathrm{Tr}\, W^k \gamma_Q + 3(\mathrm{Tr}\,\gamma_{P,L} - \mathrm{Tr}\,\gamma_{\bar{P},L}) = 4 \qquad k = 0,1,2 \tag{8.6}$$

for any of the nine fixed points in the two first complex planes. Also the number of branes and antibranes must be the same.

Using the explicit expressions for twist matrices the tadpole equations read ($j \neq i$):

$$\begin{aligned}
n_0^i - 2m_0^i = & \; -q_0^j + 2p_0^j &= 4 - N_2 - N_3 - N_4 &\quad (8.7)\\
n_1^i - 2m_1^i = & \; -q_1^j + 2p_1^j &= 4 - N_2 - N_3 - N_1 &\quad (8.8)\\
n_2^i - 2m_2^i = & \; -q_2^j + 2p_2^j &= 4 - N_2 - N_1 - N_4 &\quad (8.9)
\end{aligned}$$

where we have used that $N_0 + N_1 + N_2 + N_3 + N_4 = 8$. The total gauge group is (when all branes are at fixed points) thus

$$Sp(2N_0) \times \prod_{s=1}^{4} U(N_s) \times \prod_{a,i,j\neq i}^{2} [SO(2m_a^i) \times U(n_a^i)] \times [SO(2p_a^j) \times U(q_a^j)] \tag{8.10}$$

The fermionic spectrum which is supersymmetric on the branes is given by

$$QQ: \qquad 3[(2N_0, N_2) + (\bar{N}_1, N_3) + (N_1, N_4) + (\bar{N}_3, \bar{N}_4) + \Box\!\Box_{U_{N_2}}] \tag{8.11}$$

$$PP_{L_a}: \qquad 3(2m, n) + 2(1, \begin{smallmatrix}\Box\\ \Box\end{smallmatrix}) + (1, \Box\!\Box) \tag{8.12}$$

$$\begin{aligned}
QP_{L_0}: fermions- \quad &: \\
& (2N_0, n_0^i) + (\bar{N}_1, n_0^i) + (N_1, n_0^i) + (\bar{N}_2, \bar{n}_0^i) + (\bar{N}_3, \bar{n}_0^i) \\
+ \quad & (\bar{N}_4, \bar{n}_0^i) + (N_2, 2m_0^i) + (N_3, 2m_0^i) + (N_4, 2m_0^i)
\end{aligned}$$

$$\begin{aligned}
QP_{L_1}: fermions- \quad &: \\
& (2N_0, n_1^i) + (\bar{N}_1, \bar{n}_1^i) + (\bar{N}_2, \bar{n}_1^i) + (N_3, \bar{n}_1^i) + (\bar{N}_4, n_1^i) \\
+ \quad & (N_4, n_1^i) + (N_1, 2m_1^i) + (N_2, 2m_1^i) + (\bar{N}_3, 2m_1^i)
\end{aligned}$$

$$\begin{aligned}
QP_{L_2}: fermions- \quad &: \\
& (2N_0, n_2^i) + (N_1, \bar{n}_2^i) + (\bar{N}_2, \bar{n}_2^i) + (N_3, n_2^i) + (\bar{N}_3, n_2^i) \\
+ \quad & (N_4, \bar{n}_2^i) + (\bar{N}_1, 2m_2^i) + (N_2, 2m_2^i) + (\bar{N}_4, 2m_2^i)
\end{aligned}$$

In an analogous way we can also compute the non-supersymmetric massless spectrum for the anti-branes sectors. We find:

$$\begin{aligned}
\bar{P}Q_{L_0}: fermions+ \quad &: \\
& (2N_0, q_0^i) + (\bar{N}_1, q_0^i) + (N_1, q_0^i) + (\bar{N}_2, \bar{q}_0^i) + (\bar{N}_3, \bar{q}_0^i) \\
+ \quad & (\bar{N}_4, \bar{q}_0^i) + (N_2, 2p_0^i) + (N_3, 2p_0^i) + (N_4, 2p_0^i) \\
scalars \quad &: \\
& (2N_0, 2p_0^j) + (\bar{N}_1, 2p_0^j) + (N_1, 2p_0^j) \\
& +[(\bar{N}_2, q_0^j) + (\bar{N}_3, q_0^j) + (\bar{N}_4, q_0^j) + h.c]
\end{aligned}$$

$$\begin{aligned}
\bar{P}Q_{L_1}: fermions+ \quad &: \\
& (2N_0, q_1^i) + (\bar{N}_1, \bar{q}_1^i) + (\bar{N}_2, \bar{q}_1^i) + (N_3, \bar{q}_1^i) + (\bar{N}_4, q_1^i) \\
+ \quad & (N_4, p_1^i) + (N_1, 2p_1^i) + (N_2, 2p_1^i) + (\bar{N}_3, 2p_1^i) \\
scalars \quad &: \\
& (2N_0, 2p_1^j) + (\bar{N}_4, 2p_1^j) + (N_4, 2p_1^j) \\
& +[(\bar{N}_1, q_1^j) + (\bar{N}_2, q_1^j) + (\bar{N}_3, q_1^j) + h.c]
\end{aligned}$$

$$
\begin{aligned}
\bar{P}Q_{L_2} : fermions+ \quad &: \\
& (2N_0, q_2^i) + (N_1, \bar{q}_2^i) + (\bar{N}_2, \bar{q}_2^i) + (N_3, q_2^i) + (\bar{N}_3, q_2^i) \\
+ \quad & (N_4, \bar{q}_2^i) + (\bar{N}_1, 2p_2^i) + (N_2, 2p_2^i) + (\bar{N}_4, 2p_2^i) \\
scalars \quad &: \\
& (2N_0, 2p_2^j) + (N_3, 2p_2^j) + (\bar{N}_3, 2p_2^j) \\
& +[(N_1, q_2^j) + (N_2, q_2^j) + (N_4, q_2^j) + h.c]
\end{aligned}
$$

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